[TH-06] SLOPE STABILITY ANALYSIS FOR LAYERED SLOPE USING LIMIT EQUILIBRIUM HORIZONTAL SLICE METHOD

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ABSTRACT: All of the available limit equilibrium methods for slope stability analysis determine the factor of safety by dividing mass into vertical slices and deriving equilibrium equations. In this study slope in layered soil is divided into number of horizontal slices and failure is assumed to be happening on cylindrical surface, reason considering horizontal slices is that slope fails bodily in its linear dimension and the weight of horizontal slices lying above any slice will act on it and that weight plus self-weight will drive the slope failure. This condition is observed in most of the landslides where it is observed that horizontal mass movement of soil leads to failure and horizontal interslice forces play vital role in stability analysis. This paper proposes limit equilibrium equations based on force and moment equilibrium for horizontal slices for layered slope with and without considering interslice forces to derive equation for factor of safety and compare its value with well-known Fellenius method. Overall result suggests that horizontal slice method is accurate and it simulates both field conditions and slope behavior and it is needed to evaluate usability of this method for more complex problems having different types of field and failure conditions.

Keywords: Limit Equilibrium, Stability analysis, Horizontal interslice, Layered slope

1 INTRODUCTION

Layered slope is one of the most complex geometrical formation for slope stability analysis. Traditionally factor of safety is obtained by method of slices which has many approaches for calculations but when comparing complicated solutions (like Spencer (1967)) to simpler solutions (like Bishop (1955)) variations in factor of safety (FOS) is not much. Even most advanced computer solutions are found to give FOS nearer to simplest solutions. Behaviour of soil in landslides may not be predicted with certainty and with more than fourteen methods for slope stability analysis engineers have to choose one necessary solution. Method of slices are formulated based on principle of limit equilibrium which involves derivation of equations of equilibrium of forces and moments. This paper attempts to look back to very fundamental methods like Fellenius method and apply another prospective to it by dividing the slope into number of horizontal slices and deriving a solution for FOS based on force and moment equilibrium. Two layered slope are analyzed using formulated solution and result is compared with finite element method and vertical slice method.

As discussed earlier there are many methods for slope stability analysis. The primary difference among all these methods lies in which equations of statics are considered and satisfied, which interslice normal and shear forces are included, and the assumed relationship between the interslice forces. In ordinary method of slices such as Fellenius method only overall moment equilibrium is considered and no side forces are considered. Simplified Bishop’s method also satisfies overall moment equilibrium and only horizontal side forces are considered. In methods like Lowe and Karafiath, Simplified Janbu, Corps of Engineers, Modified Swedish, Janbu’s GPS procedure satisfy force equilibrium of each slices and overall force and moment equilibrium and assumptions are made regarding inclination of interslice forces like Lowe and Karafiath’s method considered average angle of slope and failure surface inclination as interslice force inclination, Janbu’s method considered horizontal interslice similar to Bishop’s method. In more refined methods like Morgenstern and Price method all the equilibrium conditions are satisfied, Spencer assumed interslice forces are inclined at an angle θ and in M-P assumed a variable relationship between vertical and horizontal interslice forces, both methods involve rigorous calculations. Spencer (1967) quoted in his paper about simplified bishop’s method not being rigorous and not satisfying all the conditions of equilibrium and still giving accurate results. So regardless of rigorousness and equilibrium conditions satisfied accuracy of LEMs might be dependent on relationship of assumed interslice forces to reality. In this study two different solutions are derived which are 2n+1 and 3n considering overall force and moment equilibrium for all slices.

2 Formulation of Horizontal slice method

2.1 Assumptions

For formulation it is assumed that failure is occurring at circular surface. The weight of soil mass is driving the failure and shear components C and Φ are the strength parameters in resisting the failure. Shear resistance due to cohesion and friction is considered tangential to failure surface and base normal reaction is passing through center of rotation of failure surface also the weight of slice act at C.G. of slice. Shear and normal forces are assumed to act at midpoint of failure side of slices. Soil mass is to be divided into number of horizontal slices and circular failure side of slice is assumed linear for simplicity of analysis. And an average value of factor of safety is to be calculated for all slices. Necessary notations for formulation are described in table 1.

Table 1 : Notations

Symbol	Description
N	Normal Force
S	Shear Force
C	Cohesion
W	Weight
Φ	Angle of internal friction
α	Angle of failure side of slice with horizontal
i	Notation representing slice number
l	Inclined length of slice on failure side
F	Factor of safety
R	Radius of circular failure surface
H	Horizontal interslice force
O	Center of rotation
a	distance of c.g. of slice from failure surface

2.2 Methodology

A circular failure surface is assumed and mass is divided into number of horizontal slices as shown in figure 2. Figure 3 shows forces acting on any individual horizontal slice i. Components of forces are as shown in figure 3. For formulation vertical and horizontal force equilibrium of an individual slices are equated and equation for base normal force is derived. To find factor of safety moment equilibrium is performed for individual slice and equation for F is derived.

2.3 Mathematical formulation without considering any interslice forces

From figure 3 taking vertical equilibrium of forces for any slice i:

$\sum V=0$

$$N_i \cos\alpha + S_i \sin\alpha - W = 0 \quad (1)$$

Considering horizontal equilibrium of forces:

$\sum H = 0$

$$N_i \sin\alpha - S_i \cos\alpha = 0 \quad (2)$$

By solving equation (1) and (2)

$$N_i = W_i / (\cos\alpha + \tan\alpha * \sin\alpha) \quad (3)$$

From definition of factor of safety according to swedish circle method,

Factor of safety = (resisting moment) / (overturning moment) (5)

Resisting moment is due to shear resistance along slip surface which is according to mohor-coulomb theory,

$$S = (c*l + N\tan\varphi)$$

And driving moment is due to weight

$$D = w*(R\sin\alpha - a_i)$$

Where a_i = distance of c.g. of slice from failure surface as shown in figure 4.

So factor of safety will be given by following equation

$$F = (C*l_i + N_i \tan\o)*R / (W_i*(R\sin\alpha - a_i))$$

Average FOS for all slices can be calculated by Summing up for all slices as,

$$F = (\sum C*l_i + \tan\o * \sum N_i)*R / \sum(W_i*(R\sin\alpha - a_i)) \quad (6)$$

Equation (3) can be used to calculate N and equation (6) can be used to calculate factor of safety.

Following table 2 and table 3 contains list of unknown conditions and equilibrium equations considered in above formulation accordingly. From table it can be seen that HSM without interslice formulation is (2n+1) formulation.

Table 2: List of unknown

Number	Unknown
n	Base normal force N
n	Shear force S
1	Factor of Safety
Total =	2n + 1

Table 3: List of equilibrium equation

Number	Equilibrium condition
n	Vertical force equilibrium
n	Horizontal force equilibrium
1	FOS definition as per eq. (5)
Total =	2n + 1

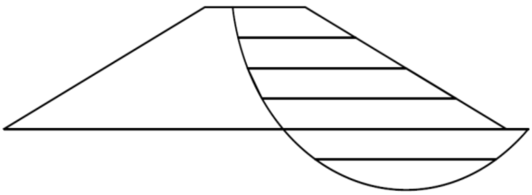

Figure 2: Horizontal slices in a typical slope

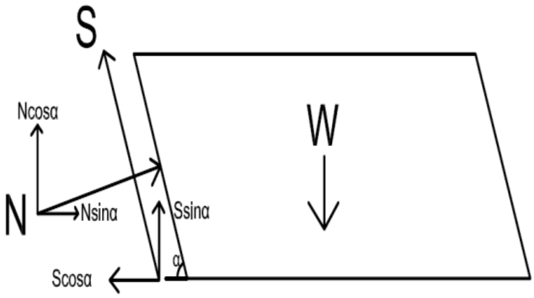

Figure 3: Forces acting on i^{th} slice

2.4 Mathematical formulation considering interslice forces

Assumptions in section 2.1 remain same for following formulation with additional assumption of horizontal interslice force. Major benefit of Horizontal Slice Method is that there is no need to consider interslice force acting in vertical direction. Figure 5 shows forces acting on a horizontal slice with two horizontal interslice forces acting in opposite direction. Here H_i and H_{i+1} are horizontal interslice force acting on any slice i.

Taking vertical force equilibrium following equation for N can be derived: (s = cl + Ntanφ, similarly to previous section is used)

$$N = (W_i - C^*l_i^*\sin\alpha/F) / (\cos\alpha+\sin\alpha^*\tan\phi/ F) \quad (7)$$

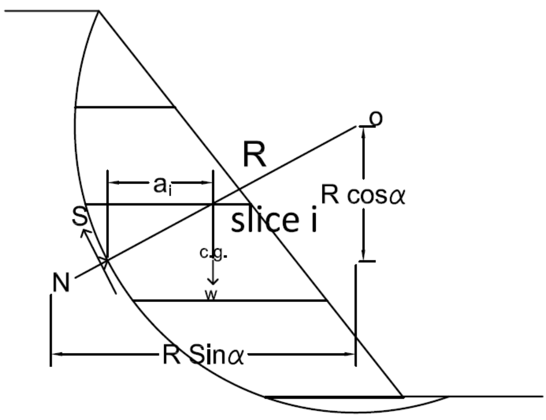

Figure 4: Diagram showing distance between center of rotation to forces

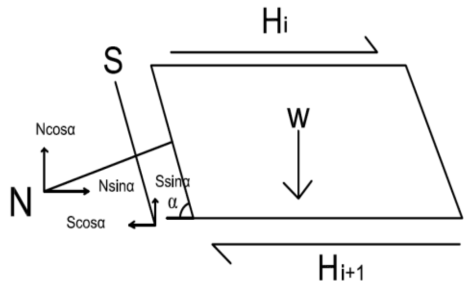

Figure 5: Forces acting on i^{th} slice including horizontal interslice force

Value of F can be assumed initially and N can be calculated. Now taking horizontal force equilibrium

$$\Delta H_i = \{(C^*l_i+N_i\tan\phi)/F\}^*\cos\alpha - N\sin\alpha \quad (8)$$

Where $\Delta H_i = H_i - H_{i+1}$

Now referring to figure 4 and taking moment equilibrium at center of rotation O

$$W_i^*(R\sin\alpha - a_i)+((C^*l_i+N_i\tan\phi)/F)^*R+\Delta H_i^*R\cos\alpha=0$$

N is passing through center so it will not create any moment at center. Here ΔH_i is assumed to be acting at c.g. of slice for simplifying the solution but two different horizontal interslice forces can be considered with different distances form O, it will give solution with a little more accuracy. By equating above formula F can be equated as,

$$F = ((C^*l_i+N_i\tan\phi)^*R)/((R\sin\alpha - a_i)^*W_i)+\Delta H_i^*R\cos\alpha)$$

Average F for entire slope can be calculated by summing up for all slices as,

$$F = ((\sum C^*l_i + \sum N_i\tan\phi)^*R) / (\sum(R\sin\alpha - a_i) ^*W_i) + \sum\Delta H_i^*R\cos\alpha) \quad (9)$$

Value of ΔH and N are calculated using (7) & (8), initial value of F is assumed and new value of F is calculated using equation (9). This procedure becomes iterative and is repeated till F is converged to a particular value.

Following table 4 and table 5 contains list of unknown conditions and equilibrium equations considered in above formulation accordingly. From table it can be seen that HSM with considering interslice formulation is (3n) formulation.

Table 2: List of unknown

Number	Unknown
n	Base normal force N
n	Shear force S
n-1	Horizontal interslice force
1	Factor of Safety
Total =	3n

Table 3: List of equilibrium equation

Number	Equilibrium condition
n	Vertical force equilibrium
n	Horizontal force equilibrium
n	Moment equilibrium of forces
Total =	3n

3 Validations

For validation of equation (6) and equation (9) a Layered slope problem chosen as shown in figure 7 (Arora, 2004). Slope is divided into six horizontal and vertical slices and shear parameters and unit weight is considered according to type of soil involved into the slice. Table 4 represents comparison of FOS with respect to Fellenius method based on vertical slices and HSM.

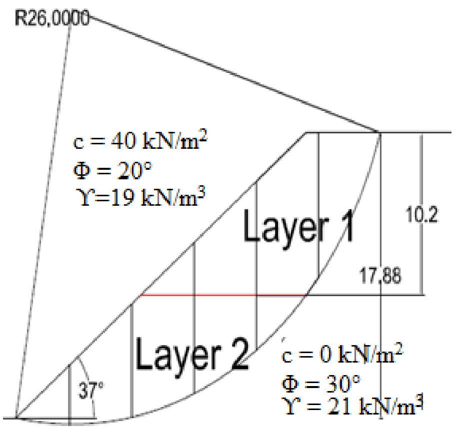

Figure 5: Typical example for validation

Table 4: Comparison of FOS

Method	Factor of safety
Fellenius (using 6 vertical slices)	1.134
HSM (no interslice, with 6 horizontal slices)	1.022
HSM (considering interslice, with 6 horizontal slices)	0.972

4 CONCLUSIONS

In case of non-homogeneous soil deposits HSM method seems to be accurate as weight components is included cumulative order for each layer which is true simulation of any soil movement observed in case of landslide failure or bearing capacity failure. Proposed formulation for FOS considering interslice forces it is observed that resultant force in force polygon stays in same direction and has less effect from horizontal interslice force which is not the case in vertical slice methods. In HSM considering horizontal interslice force procedure becomes iterative and it is very easy comparing to rigorous vertical slice methods which considers both horizontal and vertical interslice forces which may not be responsible for failure. Illustrative example taken for validation shows that factor of safety is much closer to VSM but in case of non-homogenous soils or in other words layered soils HSM estimates lower FOS considering same limit equilibrium approach applied to all methods. When horizontal interslice forces were considered FOS was converged after 62 iterations and it gives slightly lower factor of safety. Here assumptions regarding interslice forces are made to evaluate accuracy of method and not to increase equilibrium conditions. It is suggested for researchers to do in depth evaluation of advantages and disadvantages of horizontal slice method and add a different prospective to limit equilibrium procedures. When analyzing slope using two-dimensional vertical slice methods it is not possible to calculate horizontal force component in third dimension, it may be well incorporated when analyzing slope using horizontal slices.

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